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CALCULATING BASAL TEMPERATURES IN ICE SHEETS: AN EXCEL SPREADSHEET METHOD

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ABSTRACT

The flow of ice sheets and their geomorphological impact is greatly influenced by their basal thermal regime. Calculations of basal temperatures in ice sheets are therefore fundamental in evaluating glacier dynamics and in determining the spatial distribution of zones of erosion and deposition beneath ice masses. Calculations of basal temperatures are not frequently attempted, however, primarily because of the techniques required to solve the heat conduction equation between the ice surface and the base. This paper describes a new Excel spreadsheet method of solving this equation that can readily be applied to both former and contemporary ice sheets. The application of the spreadsheet is illustrated with two examples. The first provides a calculation of basal thermal regime beneath the north eastern part of the Scottish ice sheet during the last glacial maximum; the second shows how basal ice temperatures can be calculated beneath the modern Antarctic ice sheet. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: ice sheets; basal thermal regime; spreadsheet; Scotland, Antarctica

INTRODUCTION

The basal ice temperature (the basal thermal regime) of an ice sheet is of fundamental importance to three areas of glaciology:

1. it is critical to mechanisms of glacier flow, controlling the extent and location of areas of basal sliding and potential for subglacial sediment deformation (Boulton and Hindmarsh, 1987; Alley, 1989; Clark and Walder, 1994);
2. it controls the production of meltwater and the areas in which subglacial water may be stored, for example, in subglacial lakes (Siegert *et al.*, 1996; Siegert and Glasser, 1997; Siegert, 2000);
3. it is one of the primary controls on the level of geomorphological activity including zones of erosion/protection beneath ice sheets (Sugden, 1977; Gordon, 1979; Kleman, 1994; Glasser, 1995; Kleman *et al.*, 1997; Kleman and Hättestrand, 1999).

To calculate basal temperatures in an ice sheet an equation must be solved that details the heat conduction between the ice surface and the base. The solution involves the use of two infrequently used tabulated functions. As a result of this uncommon procedure, relatively few calculations of ice sheet thermal regime have been performed outside of ice sheet modelling experiments. In this paper we introduce an Excel spreadsheet

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method for calculating basal ice temperature along a one-dimensional flowline of an ice sheet. Two spreadsheets are described: one that can be used to calculate basal thermal regime for a sector of a former Scottish ice sheet (SCOTLAND.XLS) and one that can be applied to calculations of basal thermal regime beneath the modern Antarctic ice sheet (ASTROLABE.XLS).

NUMERICAL MODEL

The spreadsheet calculates basal ice sheet temperatures using the numerical model of Budd *et al.* (1971), in which a heat conduction equation is solved through a series of vertical columns of ice along an ice flowline.

The basal ice temperature T_b is given by:

$$T_b = T_s - H \left[Y_b \frac{\text{erf}(y)}{y} - Y_s 2E(y) \right] \quad (1)$$

$$y = \sqrt{\left(\frac{bH}{2k} \right)} \quad (2)$$

$$Y_b = \frac{\Lambda g}{K} + \frac{\tau_b u}{K} \quad (3)$$

$$Y_s = \frac{u\alpha\lambda}{b} \quad (4)$$

where T_s is the surface temperature ($^{\circ}\text{C}$), H is the ice thickness (m), Y_b is the total heat flux at the ice sheet base (W m^{-2}), Y_s is the heat gradient at the ice sheet surface ($^{\circ}\text{C m}^{-1}$), Λg is the geothermal heat flux (W m^{-2}) and $\text{erf}(y)$ is the error function. The basal shear stress is τ_b (Pa), b is the surface accumulation (m weq a^{-1} ; weq = water equivalent), α is the surface slope of the ice sheet (in radians), K is the thermal conductivity of ice ($2.1 \text{ W m}^{-1} \text{ K}^{-1}$), λ is the altitudinal air-temperature lapse rate (assumed to be held constant at $0.007^{\circ}\text{C m}^{-1}$, following Diamond (1960)) and k is the thermal diffusivity of ice ($36.29 \text{ m}^2 \text{ a}^{-1}$). The balance velocity u is given by:

$$u = \frac{b(\chi)\chi}{H} \quad (5)$$

where χ is the distance from the ice divide and $b(\chi)$ is the mean accumulation over that distance. The function $E(y)$ is given by:

$$E(y) = \int_0^y F(\xi) d\xi \quad (6)$$

where $F(\xi)$ is a function known as the Dawson integral. Both the error function ($\text{erf}(y)$) and the Dawson integral are tabulated functions (Abramowitz and Stegun, 1965). In addition, Rybicki (1989) provides a numerical recipe for the Dawson integral that can be readily used in a spreadsheet.

The simplest way to use the spreadsheet is to calculate basal ice temperatures along a one-dimensional ice sheet flowline. Calculations of basal thermal regime are also possible in two dimensions by running the spreadsheet across a number of neighbouring flowlines.

MODEL ASSUMPTIONS

The model assumes five glaciological conditions. The first is that the ice sheet is in steady state. This means that volumes of ice accumulation and ablation match exactly, and the flux of ice within the ice sheet acts to maintain the surface profile. The second condition is that heat conducted transverse to ice flow is negligible in comparison to that conducted along the flow line. Third, the advection of ice is assumed to occur in the

vertical direction only. The advection rate is also assumed to be constant and equal to the rate of surface warming. The fourth condition is that all friction is assumed to apply at the base of the ice sheet. Fifth, the thermal diffusivity of ice (k) and the strain rate of ice are assumed to be constants. The rationale behind these assumptions and the full derivation of the fixed-column model are discussed in detail by Budd *et al.* (1971). The basic calculations have been tested against modern ice masses such as the Antarctic ice sheet and proved capable of predicting accurately measured basal ice temperatures (Budd *et al.*, 1971).

USING THE SPREADSHEETS

The spreadsheets (SCOTLAND.XLS for former ice sheets and ASTOLABE.XLS for current ice masses) are both available from the Wiley Interscience website (www.interscience.wiley.com). They have been developed using Microsoft Excel 97 and are known to work with Excel 2000. They are not compatible with earlier versions. Some versions of Excel will require the user to install the mathematical functions used in the calculations. To do this, run the Setup program to install the Excel Analysis ToolPak. After the Analysis ToolPak has been installed, it must be enabled using the **Add-Ins** command on the **Tools** menu. The spreadsheets consist of five worksheets. The first worksheet is the only one in which the user is required to input any data. Worksheets 2–5 automatically perform the integration required to solve the heat conduction equation. There is no need to alter any of the values in worksheets 2–5. The assumptions used in the calculations mean that both spreadsheets are suitable only for calculations of thermal regime at the ice sheet scale. Since some of the assumptions do not hold for smaller ice masses, the spreadsheet cannot be used to predict basal temperatures at the valley glacier scale.

The first step in using either spreadsheet is to establish values for the various input parameters needed to perform the calculations. Some of these values require knowledge of the spatial geometry of the ice sheet (such as the position of the ice margin and ice divide), whilst others require knowledge of environmental conditions (such as accumulation rate and temperature at the ice sheet margin). For simplicity, here we describe only the operation of the Scottish ice sheet spreadsheet. Changes to the spreadsheet for contemporary ice sheets are described later. The input values needed (with units and column letters used in the spreadsheet) are as follows:

1. Distance from the ice sheet margin (m) (column A)
2. Distance from the ice divide (m) (column B)
3. Ice surface elevation (m) (column C)
4. Bed elevation (m a.s.l.) (column D)
5. Temperature at the ice sheet margin ($^{\circ}\text{C}$) (column G)
6. Altitudinal air-temperature lapse rate ($^{\circ}\text{C m}^{-1}$) (column H)
7. Accumulation rate (m weq a^{-1}) (column K)
8. Geothermal heat flux (W m^{-2}) (column L)

These columns are highlighted in the spreadsheet to show where the user can alter the input parameters. When these data are entered into the spreadsheet, it automatically calculates the basal temperature at each point along the ice sheet flowline. The number of points along the flowline can be specified in the spreadsheet simply by increasing or decreasing the number of rows and copying cells as appropriate. The output, highlighted in column T, is the basal temperature along the flowline in degrees centigrade. Temperatures above the pressure melting point are notional since above this temperature all excess heat is used to melt basal ice, leaving the basal temperature at the pressure melting value.

The spreadsheet SCOTLAND.XLS is currently configured to calculate basal temperatures at 10 km intervals along a flowline of a former Scottish ice sheet at the last glacial maximum (Gordon and Sutherland, 1993). The margin of this ice sheet is assumed to lie in the North Sea and the ice divide is located over Rannoch Moor. This produces an ice sheet with a half width of approximately 190 000 m (columns A and B). The ice surface elevation (column C) has been reconstructed using a parabolic ice sheet profile with a basal shear stress of 100 kPa, constructed inland from the ice margin using the formula of Nye (1952). The bed elevation

(column D) is the height of the present-day topography taken from UK Ordnance Survey maps of the area. The spreadsheet uses these inputs to calculate a notional ice thickness in column E, before determining an isostatically adjusted ice thickness based on an ice/Earth mantle density conversion factor of 1.36 in column F. The ice surface temperature along the flowline is calculated in column I from the value entered for the temperature at the ice sheet margin (column G) and the altitudinal lapse rate of $0.007^{\circ}\text{C m}^{-1}$ (column H). The result is an ice surface temperature that decreases up-ice with altitude (column I). The user may change any of these values to reflect the palaeoenvironmental conditions in other geographical settings.

Ice surface gradients are calculated in the spreadsheet in column J, using the average gradient over 10 km. The value for ice surface elevation in cell C21 is duplicated in cell C22 to allow the calculation of the ice surface gradient at the ice divide. This results in an ice surface gradient of 0.00 (cell J21) at the divide, which is a reasonable approximation for large ice masses. Further inputs are required for the ice accumulation rate in metres per year, (column K) and the geothermal heat flux (column L). In the case of the Scottish ice sheet the accumulation rate has been estimated at 0.75 m a^{-1} . The user can vary the accumulation rate to take account of different geographical settings such as location relative to atmospheric moisture sources. The geothermal heat flux has been set to a value representative for northern Scotland (0.05 W m^{-2}) (Robin, 1955).

The spreadsheet uses these values to calculate the balance velocity of the ice sheet (column M) and the basal shear stress at the base of each column of ice (column N). Together with the geothermal heat flux these values provide a measure of the heat applied to the base of the ice sheet (the basal thermal gradient, calculated in column O). The rate of surface warming on the ice sheet (surface temperature gradient) is calculated in column P from the balance velocity and the ice surface slope, in conjunction with the altitudinal air temperature lapse rate and ice accumulation rate. The surface temperature gradient accounts for the advection of cold firn into the ice sheet from its surface layers. The error function $\text{erf}(y)$ and Dawson integral are calculated in columns Q, R and S using the values tabulated in worksheets 2–5. The spreadsheet looks up the values of these functions automatically and there is no need for the user to adjust this part of the spreadsheet. The final step is the calculation of the basal ice temperature (in $^{\circ}\text{C}$) which the spreadsheet outputs in column T. The pressure melting point (in $^{\circ}\text{C}$) of the ice sheet, based on the ice thickness at each calculation point, is given for comparison in column U.

Sensitivity tests using the model suggest that the calculated basal temperatures are more sensitive to changes in some input parameters than others (Glasser, 1991). For example, the model is very sensitive to changes in ice thickness and therefore to changes in the ice surface profile used to represent the ice sheet. The general pattern is for warmer basal temperatures beneath thicker ice. The model is also sensitive to changes in the accumulation rate. High accumulation rates near the ice divide increase the rate at which cold firn accumulates on the ice surface and therefore decrease basal temperatures. This process is thought to be responsible for maintaining freezing conditions at the base of the Greenland ice sheet, despite it being over 3 km thick at the ice divide (Huybrechts, 1996). High accumulation rates nearer the ice margin produce higher basal temperatures because of the increased velocities and associated frictional heat production.

Since there is a direct relationship between the ice surface temperature and the basal temperature, the model is also sensitive to the mean annual temperature imposed at the ice sheet margin. The warmer the imposed ice marginal temperature, the warmer the basal temperatures. Because the spreadsheet calculates basal temperatures along a single flowline, the model does not take account of any convergence and divergence of flow due to the subglacial topography. The effects of this may be significant, especially in areas where there is high topographic contrast or a likelihood of ice stream development (Glasser, 1995; Siegert and Glasser, 1997). Finally, since the model is not time dependent (i.e. it does not calculate the change in basal temperatures over time), the results are best interpreted as a snapshot in time under steady-state conditions.

EXAMPLES OF APPLICATIONS OF THE SPREADSHEET

Example 1: a sector of a former Scottish ice sheet

The first experiment is a calculation of basal ice temperatures beneath a sector of a former Scottish ice sheet at its maximum (SCOTLAND.XLS). The inputs represent the palaeogeographical and palaeoenvironmental

conditions prevailing at this time. These are the geographical extent of the ice sheet, the ice sheet profile, the bed elevation, the atmospheric temperature regime, altitudinal lapse rate, accumulation rate, and the geothermal heat flux. The ice sheet profile is based on a 100 kPa parabolic ice surface profile. The margin of the ice sheet is set at the Bosies Bank moraine, 25 km offshore from the present-day North Sea coast of Scotland. Annual average temperature at the margin is assumed to be -9°C , with an altitudinal lapse rate of $-0.007^{\circ}\text{C m}^{-1}$. Accumulation rates are set to 0.75 m a^{-1} . Glasser (1995) has provided a full discussion of the derivation of these input parameters and the sensitivity of the model to changes in the values selected for each.

Under the steady-state conditions prescribed in this experiment, the basal ice temperatures increase steadily from the ice divide towards the margin (Figure 1). Under the ice divide and immediately to its east are frozen-bed conditions, whilst towards the margin predicted rates of basal melting are high. This is due to a combination of (i) increasing velocity towards the margin (resulting in greater ice discharge and therefore increased frictional heat production) and (ii) the relatively milder ice surface temperatures with decreasing elevation towards the margin. Note that the temperatures close to the ice margin are probably overpredictions because below the equilibrium line there would be a progressive mass loss due to ablation (not accounted for in this model). The effect of topography is also evident, with relatively cold basal temperatures where there is thin ice over topographic high points, such as the Cairngorm Mountains (Figure 1). This area of northeast Scotland is noted for its selectivity of glacial erosion (Glasser and Hall, 1997) and the pattern of basal ice temperatures can therefore be compared against field evidence of glacial erosion and protection in order to provide insight into the processes of landscape evolution beneath ice sheets (Glasser, 1995).

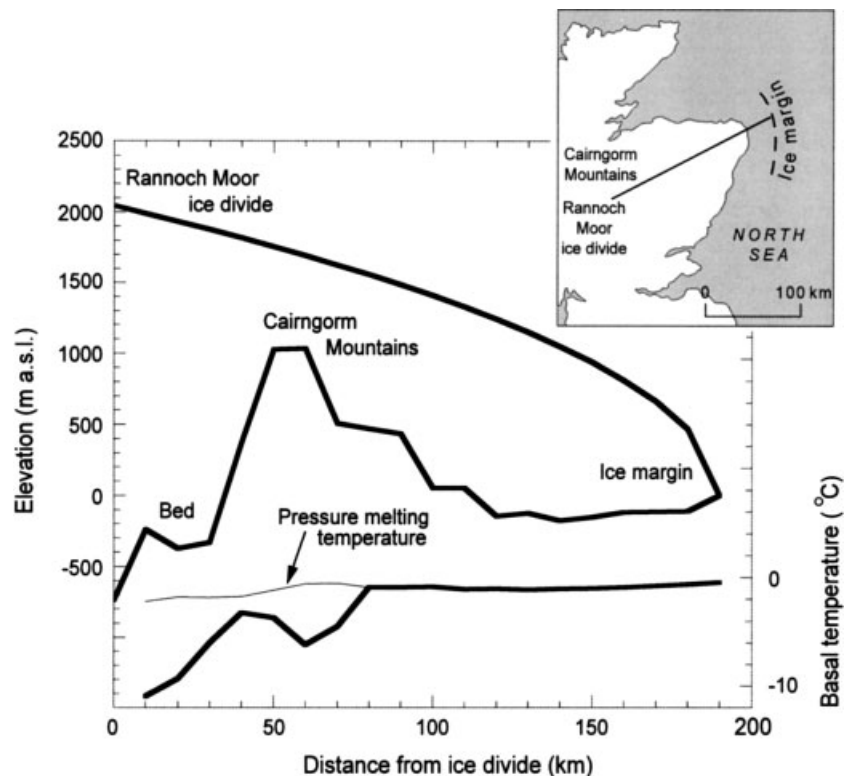


Figure 1. Reconstructed ice surface profile, basal topography and calculated basal temperatures for a sector of a former Scottish ice sheet using the Excel spreadsheet. Subglacial temperature is denoted by a thick line, and the thin line shows the pressure melting temperature. If the calculated temperature is below the pressure melting temperature then the ice sheet base is frozen. The location of the flowline is indicated on the map of Scotland (inset)

Example 2: the present-day Antarctic ice sheet

A second experiment shows how modern subglacial conditions along a flowline in East Antarctica can be evaluated (ASTROLABE.XLS). The ice thickness and surface elevation along a flowline from Dome C ice divide, through the Astrolabe subglacial basin to the ice sheet margin at Terre Adélie, was established from *Antarctica: Glaciological and Geophysical Folio* (Drewry, 1983). The rate of surface accumulation on the ice sheet along the flowline was interpolated from accumulation patterns derived from direct field measurements (Giovinetto and Bentley, 1985). The surface temperature of the ice sheet is obtained from the mean annual air temperature at the margin and the altitudinal lapse rate (Robin, 1983). Column I is therefore an input for air temperature, and Column H is a calculation of lapse rate derived from air temperature and surface elevation measurements. If field measurements of surface air temperature exist, users can input these data directly into the spreadsheet and switch off the altitudinal lapse rate. A large (30 km long) subglacial lake, identified from radio-echo sounding (Siegert *et al.*, 1996), is located at the mouth of the Astrolabe Subglacial Basin, and is evidence of subglacial melting across this region. Therefore, in this experiment, subglacial temperatures can be compared to direct evidence of subglacial melting.

The spreadsheet configuration differs from the Scottish case in three ways. Firstly, the ice sheet is assumed to be in equilibrium and there is therefore no need to adjust the model for isostatic depression. Columns E and F are therefore identical. Secondly, the rate of accumulation varies along the transect according to field measurements. This is notably different from the accumulation input used for the Scottish ice sheet, where a constant accumulation rate was assumed. Thirdly, since the accumulation rate varies along the flowline, balance flux calculations are different. Two new columns (L and M) are introduced, in which cumulative accumulation and averaged accumulation (between the cell and the ice divide) are calculated. The balance velocity is based on (1) the average accumulation upstream, multiplied by the distance to the ice divide, divided by the ice thickness at that cell, and (2) the divergence of ice that occurs from the ice divide to the margin. Ice divergence in our flowline can be treated simply because the East Antarctic ice sheet can be regarded as a symmetrical circular ice mass. In effect we are modelling a 'pie' section with a flowband width linearly increasing from 0 at the divide to the margin (Huybrechts *et al.*, 1996). This is implemented by multiplying the balance velocity by a factor 0.5. The actual correction term is $(-F/w)(dw/dx)$ where F is the flux, w is the width of the pie section and x is the horizontal coordinate.

The calculations of basal temperatures using the spreadsheet indicate two areas of basal melting: a first area in the upper part of the transect and a second area over the Astrolabe Subglacial Basin (Figure 2). These areas of basal melting are related primarily to increased ice thickness over depressions in the subglacial topography. Elsewhere in the transect, the model predicts basal temperatures below the pressure melting point. The locations of the areas of basal melting coincide with the known locations of subglacial lakes beneath the ice sheet (Siegert *et al.*, 1996; Siegert and Glasser, 1997), providing an independent test of the predicted basal thermal regime.

CONCLUSIONS

The Excel spreadsheet method presented in this paper can be used to calculate basal ice temperatures for both former and existing ice sheets. The spreadsheets can be used in three ways. First, they can be used to evaluate the modern or ancient subglacial conditions for any given set of input data. Second, they can be used to evaluate the glaciological conditions required to produce certain subglacial conditions, such as those required to match field evidence of subglacial melting or freezing. Third, they can be used to systematically alter variables in the model in order to ascertain the sensitivity of the ice sheet thermal conditions to changes in model inputs. Although the spreadsheets are set up to calculate temperature at the base of a single flowline, a series of neighbouring flowlines can be used to develop their capability into two dimensions. Here we have illustrated two different uses of this type of spreadsheet. The first of these, for a former Scottish ice sheet, demonstrates how calculations of basal temperatures in former ice sheets can be compared to geomorphological evidence of zones of basal melting/freezing. The second example, for the present-day Antarctic ice sheet, illustrates the close association between the predicted basal thermal regime and the presence/absence of subglacial lakes.

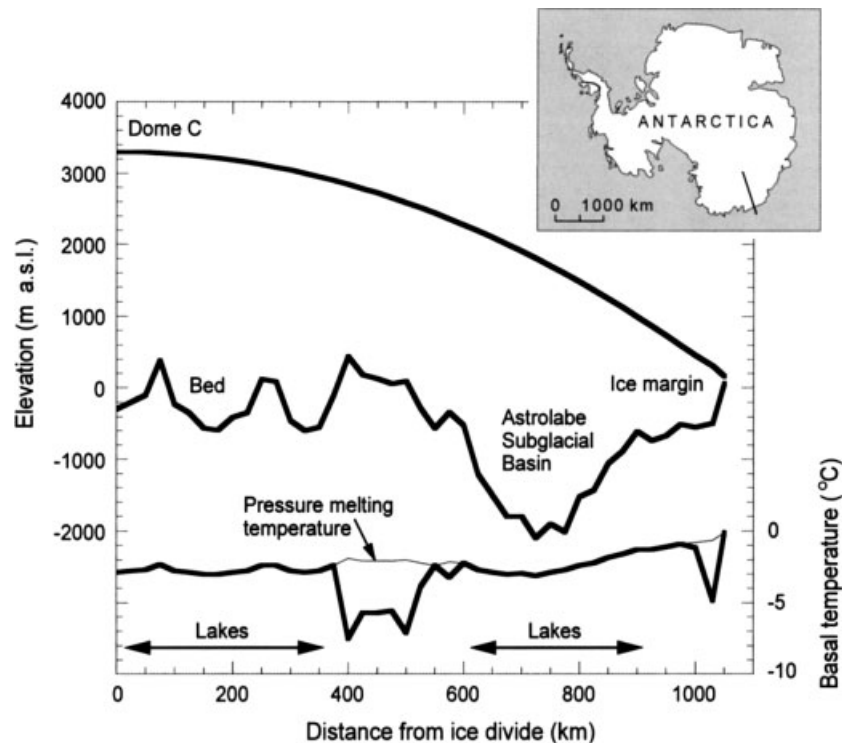


Figure 2. Ice surface, subglacial topography and calculated basal temperatures along a flowline between Dome C and Terre Adélie, East Antarctica. Subglacial temperature is denoted by a thick line, and the thin line shows the pressure melting temperature. If the calculated temperature is below the pressure melting temperature then the ice sheet base is frozen. The locations of two regions of subglacial lakes, one beneath Dome C and one across the Astrolabe Subglacial Basin, coincide well with regions calculated at the pressure melting temperature. The location of the flowline is indicated on the map of Antarctica (inset)

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