Murder, Death and Disease

Ian Eames
Email: I.Eames@meng.ucl.ac.uk
University College London, Gower Street, London WC1E 7JE

Yuguo Li
The University of Hong Kong
7/F Haking Wong Building
Pokfulam Road, Hong Kong, China

Julian W Tang
The Chinese University of Hong Kong
Prince of Wales Hospital
Shatin, NT, Hong Kong SAR, China

Julian CR Hunt
University College London, Gower Street, London WC1E 7JE

Summary

Understanding the transport of droplets in the air, introduced by foul play or accident are key processes in forensic science and the spread of infections. We illustrate these problems using a high profile murder case in the UK and airborne transmission of disease. Both problems involve trying to understand the short and long range transport and fate of droplets and particles. The transport of a fine mist of droplets (of blood or mucus) caused by coughing, breathing, sneezing or displacement of the lung cavity are largely determined by the presence of coherent vortical structures (close to the human body), evaporation (in the case of disease transmission) or mean / turbulent flows (for long range transport). The examples we introduce demonstrate how simple physical models can be used to guide our understanding of important public issues, relevant to life/death situations.

1. Introduction

The transport of a contaminant (smoke, pollutants, radioactive particulate, biohazards) has always provided a rich area for applied mathematicians to study. This draws in many different techniques (not just computations) including pattern detection or topology of the flow, sensitivity and order of magnitude estimates. There is an increasing awareness of the importance of understanding how biological material spreads either in the context of forensic science, the spread of air borne infections (SARS, bird-flu, clostridium-difficile) or the cross contamination of plant crops with genetically modified plants in adjacent fields. The aim of this article is to show why droplets are capable of being transported over significant distances. This is certainly surprising because small droplets fall very quickly to the ground in still air. The two examples we introduce are topical – the first is related to the forensic science of a high profile murder case and the second is related to air-borne transmission of disease, which has increasing prominence in the context of the hospital environment (see Tang et al 2006; Wells 1936).

To understand how particles or droplets are dispersed we need to calculate how they move. The most general approach to study how discrete elements are dispersed is to use a Lagrangian formulation where each element (droplet/particle) is followed in time using an equation of motion based on considering a balance of forces on each element. Consider a spherical droplet or particle (of diameter d, density ρ_p) moving with velocity $\boldsymbol{v}$ in an air flow $\boldsymbol{u}$. The equation of motion describing how they move with time is

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{1}{t_p}\left(\boldsymbol{u}(\boldsymbol{X},t) - \boldsymbol{v}\right) - \boldsymbol{g}, \qquad \frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} = \boldsymbol{v}. \tag{1}$$

X,Y are the horizontal and vertical position of the droplet/particle and $\boldsymbol{X} = (X, Y)$, $\boldsymbol{g} = g\hat{\boldsymbol{y}}$ is the gravitational acceleration and t_p is the response time. This model is based on a Stokes drag law, where the viscous drag is proportional to the relative slip between the droplet and air. Equation (1) is a second-order linear differential equation with a non-linear forcing caused by the air motion – note the air velocity may vary with position and time, this is explicitly described in the argument of $\boldsymbol{u}$. Particles falling in stagnant air ($\boldsymbol{u} \equiv \boldsymbol{0}$) and which do not change size, ultimately sediment with a terminal fall velocity $v_T = t_p g$. The ability of a small spherical (rigid) particle to respond to changes in the air flow is characterised by a response time

$$t_p = \frac{d^2 \rho_p}{18\mu}, \tag{2}$$

where μ is the viscosity of air, d is the droplet diameter, and ρ_p is the droplet density. The linear drag model is strictly valid for $d \leq 200$ μm, for which the Reynolds number based on the slip-velocity is less than $Re_p = dv_T/\nu < 10$. For our analysis on blood and mucus droplets, the density of the droplets is comparable to water $\rho_p = 10^3$ kg/m^3, and the viscosity of air is $\mu = 1.8 \times 10^{-5}$ Pas at 18 ^{o} C. Although since the droplets are much more viscous than air and usually contaminated (with pathogens, blood corpuscles and salt), it is realistic to also use the response time (2) for droplets.

Figure 1(a) shows the variation of the droplet fall speed v_T as a function of droplet diameter – even small droplets of diameter $d \sim 100$ μm settle rapidly with a speed of 0.3 m/s. As the size of the droplets decreases, the residence time of the droplets in the air increases significantly and with this the potential for inhalation deep into the lungs (for $d < 10$ μm). The potential for discrete elements remaining aloft in air and being transported over significant distances depends (as we can see from (1)) on both the air flow and the droplet properties. The change in the droplet properties is largely through a decrease in its diameter when evaporation is important, which in turn decreases its response time (from (2)) and fall velocity. The effect of a mean ambient flow only becomes important when the droplets are small enough (<60 μm) that the vertical air velocity $u_y(\boldsymbol{X}, t)$ is larger than the droplet fall velocity. But large droplets propelled at typical speeds are largely unaffected by air motion and move ballistically. The partition between large and small droplets is ambiguous and is typically taken to be $\sim 60\mu$m. In the context of air borne infection, droplets less than 10μm are generally referred to as droplet nuclei.

2. Murder: 158 droplets of blood

Billy-Jo Jenkins (aged 13) was beaten to death with a 50 cm tent peg while painting a patio door at home in Hastings, East Sussex on February 1997. She was found by her step-father, Sion Jenkins, who was later arrested and charged with her murder. The prosecution at Lewes Crown Court hinged on forensic evidence, which the trial judge, Mr Justice Gage called "compelling" : a fine mist of blood was found on Mr Jenkin’s jacket. Prosecution experts said the fine droplets could only have come from a fierce spray of blood at the time of the killing, proving that Jenkins was in the vicinity of Billy-Jo when she was murdered. At the trial, Jenkins' defense claimed the flecks of blood had got onto his jacket as he cradled his dying foster daughter in his arms. A paediatrician called this defense impossible because Billie-Jo would have had to have been breathing extremely hard to produce such a spray. But in a 999 call to the police Mr Jenkins said his daughter was not breathing when he found her. Another paediatrician said the distribution of blood was consistent with the wearer of the clothes delivering several blows to the head. Sion Jenkins was originally convicted after 19 days of trial in July 1998.

As later retrials would discover, there was evidence that a mentally ill man - Mr X – was in the vicinity when Billie-Jo was killed and had a fixation with pushing pieces of plastic bag up his nose; a pathologist found part of a black bin liner stuffed deeply into one of Billie-Jo's nostrils. On the day Billie-Jo died, there were numerous sightings of a psychiatric patient acting suspiciously in the park area next to the Jenkins' house. The man was arrested but released several days later.

After two retrials, Mr Jenkins was finally released after 7 years in prison. More than a dozen expert witnesses were consulted in relation to the key forensic evidence. The critical aspect which turned the case around was the strength of the scientific evidence in relation to the droplets. We analyse this evidence using simple mathematical models. There are two components to interpreting the deposition of 158 fine droplets of blood of diameter $d \sim 100\mu$m on the clothing of Mr Jenkins: (a) how the droplets were generated and (b) how they could get onto his clothing. We discuss these elements separately.

The presence of droplets of diameter $\sim$ 100 μm on clothing is usually taken as strong forensic evidence of a high-speed impact onto a pool of blood, blood sodden clothing or a body because significant energy is required to create small liquid droplets. Ballistic impacts from projectiles, such as bullets, give even finer droplets ($d \sim 10 - 50$ μm). These processes can be understood using a scaling analysis. The motion of droplets of diameter d, moving with speed v and the air/liquid interface characterised by a surface tension σ, are characterised by a dimensionless group, the Weber number: $We = v^2 d\rho/\sigma$. If droplets are created by strands of fluid being accelerated and broken up by an air flow, We has to exceed a critical Weber number $We_c \approx 10$. Droplets of diameter $d = 100$ μm, would require striking a wet fabric with a velocity $v \approx (We_c\sigma/d\rho_d)^{\frac{1}{2}}$ =3 m/s which is typical of droplets created by a blunt force trauma.

Consider now the possibility that droplets are created by the ejection of air through a small constriction (which is lined with a fluid, such as mucus or blood). Denoting the cross-sectional area at the narrowest constriction by A and the volume of air V ejected over time δt. The air velocity during this period is $u_a = V/A\delta t$. When the air flow is created by coughing through the mouth, we estimate approximate values $V = 250$ cm^3 , $\delta t \sim 1$ s, $A = 3$ cm^2 (for an orifice of diameter 1 cm). This gives an estimate of $U_a \approx 1$ m/s, close to typical values assumed for coughing but too low to be capable of generating fine droplets. The defense contended that the nose was blocked and with a build up of pressure with the air ejected through a small orifice. For a small gap of diameter 0.4 cm, the movement of a small volume of air ($V = 100$ cm^3) creates an air velocity of $u_a \sim 10$ m/s, which is typical of values assumed for a sneeze. Experiments by Prof David Denison (Royal Brompton Hospital, London) showed that Billie-Jo could indeed have inaudibly breathed out the mist, after she was dead. Tests showed that the blood could have come from tiny amounts of air leaving through a "pinhole" in Billie-Jo's nose, which was blocked with blood.

Once the droplets are generated, the next critical component is to understand how they can be transported over some distance in order to be deposited on the clothing of Mr Jenkins.

Consider a droplet ejected with a horizontal velocity v_{0X} in a uniform air stream U_x from a height H above the ground. Equation (1) reduces to

$$\frac{\mathrm{d}^2X}{\mathrm{d}t^2} = \frac{1}{t_p}\left(U_x - \frac{\mathrm{d}X}{\mathrm{d}t}\right), \qquad \frac{\mathrm{d}^2Y}{\mathrm{d}t^2} = \frac{1}{t_p}\left(-\frac{\mathrm{d}Y}{\mathrm{d}t}\right) + g. \tag{3}$$

For a droplet injected horizontally with speed v_{0X} from a height H above the ground, moves to (X, Y) in time t where

$$X = U_x t + t_p(U_x - v_{0X})(\exp(-t/t_p) - 1), \qquad Y = H - v_T t + v_T t_p(1 - \exp(-t/t_p)). \tag{4}$$

Large droplets ($t_p \gg \sqrt{H/g}$) fall to the ground in a time shorter than t_p, and by expanding (4) using a Taylor series,

$$X \approx v_{0X} t, \qquad Y \approx H - \frac{1}{2}gt^2 \tag{5}$$

This describes the ballistic motion of the large droplets, which travel in a parabolic trajectory. Large droplets strike the ground after a time $t = (2H/g)^{\frac{1}{2}}$ and move a horizontal distance $X_\infty = \sqrt{2H/g}v_{0X}$ from the point of ejection. For small droplets ($t_p \ll \sqrt{H/g}$), the effect of air drag is so large that

$$X \approx U_x t + (v_{0X} - U_x)t_p, \qquad Y \approx H - v_T t, \tag{6}$$

and droplets strike the ground at a horizontal distance

$$X_\infty \approx \frac{HU_x}{v_T} + (v_{0X} - U)t_p, \tag{7}$$

from the point of release. In the absence of an external flow, small droplets are only capable of moving a short distance $X_\infty = v_{0X}t_p$ before sedimenting to the ground. An external flow or breeze is capable of assisting in transporting small droplets much further distances. Figure 2(b) shows numerical calculations comparing the maximum horizontal distance droplets released at $H = 0.5$ m travel for the case of $U_x = 0$, 0.05 and 0.1 m/s and with an initial ejection velocity of v_{0X} =10 m/s. The height H is taken to be that of a person sitting on the ground. In the absence of a breeze, 50-100 μm diameter droplets are transported a distance 0.2-0.3 m. The effect of quite a strong breeze of $U_x = 0.1$ m/s near to the ground, assists in transporting the droplets to a distance 0.5m.

The prosecution had contented that even in the presence of a breeze, the droplets could not have been ejected from the nose or mouth and travel sufficiently far to reach Mr Jenkins. Such a breeze would have been blocked by the body of Billy-Jo and Sion Jenkins and so could not have been strong enough to carry the droplets far. Further, the maximum distance estimates correspond to the distance droplets strike the ground, and so could not travel far enough to reach Mr Jenkins via this mechanism.

An important ingredient in the physics of transport is the generation of a vortex generated by the ejection of air from the lungs. Of course air escaping from the lungs does generally not produce a vortex ring but rather a turbulent puff. Lord Kelvin established the mathematics of these self-organising and persistent flow structures by considering the integrals of the momentum equation around vortex structures. In a vortex, the air is moving vertically as well as horizontally, so that small dtoplets and particles can be suspended in moving vortices as seen in smoke rings or sand filled eddies in coastal waters. But even in a puff, vertical motion transports suspended particles over significant distances. Although droplets are ejected from vortices (because they are denser than air), when the droplets start within the vortex core, the distance they can be transported is significant and this distance increases as the size of the droplets decreases. Hunt etal (2007) studied numerically the transport of droplets and particles which are released within a propagating vortex. The salient features of their results are that the vortex initially moves quickly ($\sim 1-2$ m/s) and that the droplets will be ejected from the vortex after a time of about 0.5-1 s, traveling horizontally about 1.5m. These distances are sufficiently far to explain how the 158 droplets, over a small area, could have been found on the clothing of Mr Jenkins. This is the second essential component which was missing from the analysis by the expert witnesses from the prosecution, but picked up by the defense.

3. Disease: Airborne transmission

Pathogens can be transmitted by air from a source to a person resulting in infection (with or without it being expressed). If the pathogen has some part of its life cycle in the respiratory tract, it is more likely to be present in aerosols generated by breathing, talking, coughing and sneezing. For truly airborne pathogens (tuberculosis, measles and varicella zoster virus), the route of acquisition and dissemination of infectious particles are via the respiratory tract. In the other pathogens, acquisition of the infection is also via the respiratory tract but pathogens can enter the air by other methods (eg changing bed sheets, cleaning) and may also be ingested. There are many reported cases where projectile vomiting splashing on the floor or toilet basin has been identified as the mechanism responsible for creating aerosolised pathogen air-borne droplets, for instance, the Norwalk or "winter vomiting virus" which plagued the UK during the winter of 2007. The SARS epidemic in 2003 also revealed environmental factors that might also contribute in producing virus-laden aerosols such as those produced by nebulisers, tracheostomies, bronchoscopies, and in the Amoy Gardens outbreak, a defective sewage system.

A cough or sneeze can generate up to 3000 and 40000 droplets respectively (see figure 2(a)) (Cole & Cook 1998). Once infectious droplets are generated, the main factors that determine how they move are their size and the airflow patterns which carry them. The key to such droplets remaining in air is their evaporation which reduces their size and fall velocity. We extend the analysis of Wells (1936, 1955), to include both the settling of droplets and advection by a horizontal current U_x. Consider a droplet of initial diameter d_0, projected horizontally with a velocity v_{0X} a height H above the ground. The diffusive loss of material due to evaporation is adequately described by an isothermal diffusive process where $\mathrm{d}d^2/\mathrm{d}t = -D_e$, or by integrating gives $d = d_0(1-t/t_e)^{\frac{1}{2}}$. For unsaturated air at 18^o, $D_e = d_0^2/t_e \sim 6.1 \times 10^{-9}$ m^2/s. The evaporation time t_e is much longer than the response time ($t_e/t_p \sim 5.3(d_0/d)^2 > 5.3$), so that droplets sediment with a fall velocity based on the instantaneous droplet diameter. This is solved subject to the initial conditions $\mathrm{d}X/\mathrm{d}t = v_{0X}$, $\mathrm{d}Y/\mathrm{d}t = 0$, $X = 0$, $Y = H$ at $t = 0$. The solution to (5) is

$$X = U_x t + \frac{(v_{x0} - U_x)t_e}{1 + t_e/t_{p0}} \left(-(1 - t/t_e)^{1+t_e/t_{p0}} + 1\right), \qquad Y = H - v_{T0}(t - t^2/2t_e). \tag{6}$$

In practice, the droplets will consist of a small amount of pathogens so that the droplet mass does not tend to zero. Saliva contains a small fraction (typically $R = 1.8\%$, Duiguid 1966) of solid particulate which does not evaporate. We include this information in our analysis by assuming that after $t/t_e = 1 - R^{2/3}$, the droplet diameter is assumed to be fixed at $d/d_0 = R^{1/3}$ when only the residual material is left. After a time $t/t_e = 1 - R^{2/3}$, the droplet has fallen to $H - v_{T0}t_e(1/2 - R^{2/3}/2 - R^{4/3}/2) \approx H - v_{T0}t_e/2$. When $H > v_{T0}t_e/2$ (or $d_0 < 6(HD_e\mu/g\rho_d)^{\frac{1}{2}}$), the liquid has evaporated leaving behind a small particle containing pathogens, after which it sediments to the ground very slowly with a terminal fall velocity $\sim v_{T0}R^{\frac{2}{3}}$. The droplet moves a horizontal distance

$$X_\infty \sim \frac{(H - v_{T0}t_e/2)U_x}{v_{T0}R^{\frac{2}{3}}},$$

before it strikes the ground. When $d_0 > 6(HD_e\mu/g\rho_d)^{\frac{1}{2}}$, the droplet strikes the ground at a time t/t_e where $t/t_e \sim 1 - \sqrt{1 - 2H/v_{T0}t_e}$. The maximum distance the droplet moves is $U_x t_e(1 - \sqrt{1 - 2H/v_{T0}t_e})$.

Figure 2(b) shows the variation of the maximum distance a droplet for two initial horizontal droplet velocities ($v_{0X} = 2$ and 12 m/s) corresponding to a cough and sneeze respectively, for the case of a stagnant ambient flow and a breeze $U_x = 0.05$ m/s. The corresponding results for non-evaporating droplets is also plotted. From these calculations, we see that droplet evaporation has a negligible effect of the distance traveled for droplets greater than about 170 μm. A more detailed analysis, shows that above 500μm, the distance travelled is independent of the droplet diameter because the mode of propagation is ballistic. Below a diameter of 120 μm, the permanent distance transported is dominated by the speed of the breeze because the initial droplet velocity relative to the breeze decreases very rapidly. Evaporation clearly assists in transporting material over a significant distance, as can be seen by contrasting the evaporating/non-evaporating curves.

A cough and sneeze tend to eject droplets with an average speed of 2 and 12 m/s respectively. The major difference between a cough and sneeze is the mass of ejected material and the size distribution. A sneeze generates a distribution of droplet sizes (see figure 2a), with typically the majority (by number) in the range 10-50 μm. The size distribution for droplets generated by coughing is similar, but a much larger number of droplets and a tendency to produce many smaller droplets. Droplets larger than 500 μm are transported a distance of about 2.5-5m by a standing person ($H = 1.5$m) and this is independent of d. Droplets smaller than 170 μm will evaporate sufficiently that they remain in the air for a long time and be transported a significant distance by a mean flow.

Although the potential for travel increases significantly as the initial droplet diameter decreases, the chance of that an individual droplet contains a pathogen also decreases. Duiguid (1966) estimated that 1ml of saliva from an infected person will contain about $N_b = 1,000,000$ pathogens, and studied the probability of infection. The probability that a droplet of diameter d will not contain a pathogen is $\exp(-\lambda)$ where $\lambda = \pi N_b d^3/6 \times 10^6$, so the percentage of droplets of diameter d that contain a pathogen is $P(d) = 100(1 - \exp(-\lambda))$. While a large number of fine droplets are created by coughing and sneezing, the fraction of droplet nuclei which contain a pathogen rapidly diminishes with droplet size.

4. Conclusions

With an increasing emphasis on public health, disease transmission (particularly in hospital wards) and the spread of the pollen from genetically modified pollen, it is becoming more important to understand the potential for discrete elements (particles/droplets) being transported over large distances. We have used simple physical models which emphasised the potential for droplets and particles being transported over significant distances, ranging from a meter (due to their entrainment by vortices), to a few meters by sneezing and from ten-to-hundreds of meters due to the evaporation of small droplets. A major research challenge is how to use this information to improve the design of our environment so that the potential for air-borne infection is reduced, particularly in hospitals.

Acknowledgement Discussions with an expert witness for defense and research of the news website (www.bbc.co.uk) assisted with the preparation of this paper.

References

Cole, E.C. & Cook, C.E. 1998 Characterisation of infectious aerosols in healthcare facilities: an air to effective engineering controls and preventative strategies. *Am. J. Infection Control.* **26**, 453–464.

J.P. Duguid 1946 The size and duration of air-carriage of respiratory droplets and droplet-nuclei. J. Hyg. 4: 471-480

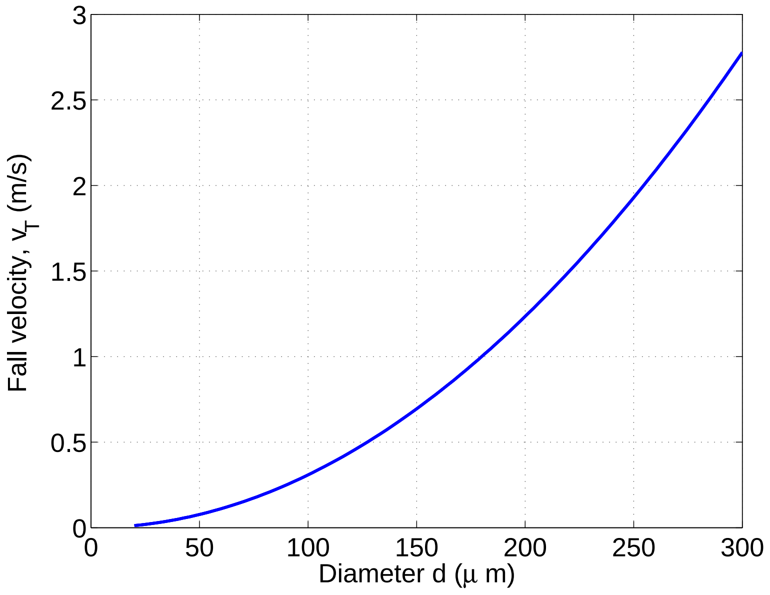
Hunt, JCR, Delfos, R., Eames, I. & Perkins, RJ. 2007 Vortices, complex flows and inertial particles. Flow Turbulence and Combustion. 79, 207-234.

Papineni, R.S., Rosenthal, F.S. 1997 J. Aerosol Med. Size distribution of droplets.

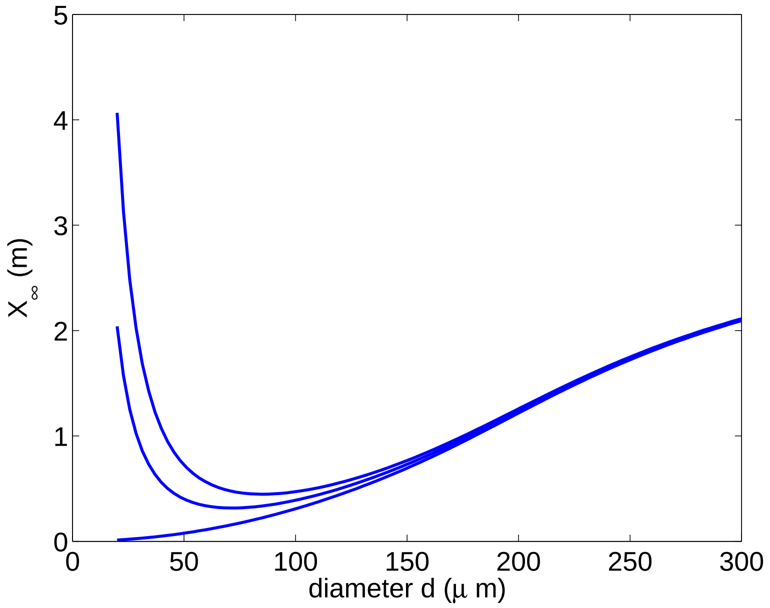
Tang JW, Li Y, Eames I, Chan PK, Ridgway GL. 2006 Factors involved in the aerosol transmission of infection and control of ventilation in healthcare premises. J Hosp Infect. 64(2):100-14.

Wells, WF 1936 On air-borne infection. Study II: Droplets and droplet nuclei. 611-618.

Wells, W.F. 1955 Airborne contagion and air hygiene. Cambridge: Havard University Press.



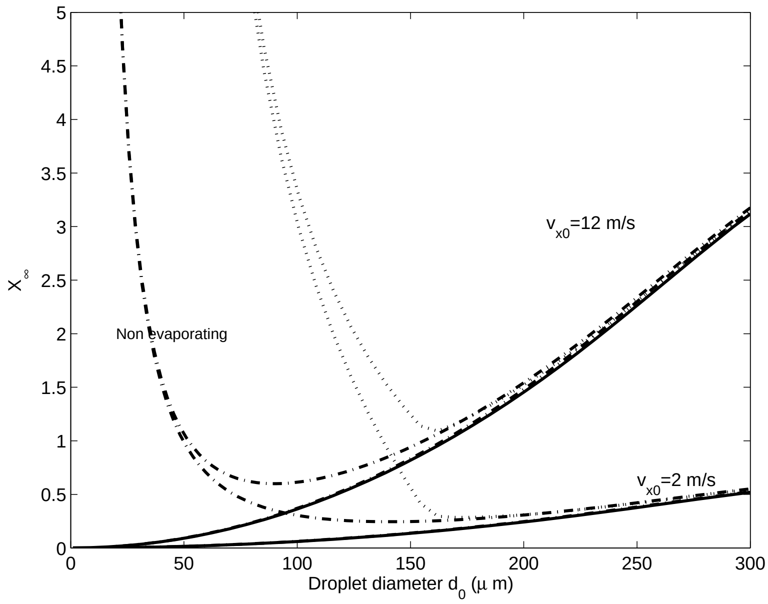
(a)



(b)

Figure 1: (a) Terminal fall velocity of droplets as a function of droplet diameter. (b) The maximum distance droplets projected horizontally at a height $H = 0.5$ m above the ground with a horizontal velocity of 3 m/s. The ambient free stream velocity is U_x=0, 0.05 and 0.1 m/s.

(a)



(b)

Figure 2: (a) Sneeze showing the generation of an array of droplets. Large droplets travel balliastically while small draplets are transported by the vortex. (b) Maximum horizontal distance droplets are transported by droplets ejected with a horizontal velocity $v_{0X} = 3, 6$ and 9 m/s, at a height of $H = 1.5$m. There is an ambient uniform flow of speed $U_x = 0.05$ m/s.