The importance of thin models – a workshop on critical regions in geophysical flows

Introduction

A **characteristic feature of large scale geophysical** as well as engineering flows is the ubiquity of thin elongated layers (whose lengths L are much greater than their thicknesses d) in which there are marked gradients and sometimes extreme values of the flow variables V, as sketched in Fig 1. They have interesting mathematical properties, since their internal structure may have random, time varying forms and locations as well as being quite complex geometrically.

Stimulated by new measurements and physical concepts, and a growing realization about their significance in planetary scale geophysical processes, there have been new developments to advance the mathematical and computational modelling of these layers. This was the context for the LIMS/ CPOM workshop on June 21 and 22, 2006, held at UCL in collaboration with IMA and Arizona State University, USA. This workshop follows on from a LIMS conference in 2004 on turbulence and waves in stratified atmospheric shear flows, funded by the US European Office of Aerospace Research and Development.

The meeting was reminded of the great contributions of Sir James Lighthill and Prof Keith Stewartson of UCL who were pioneers in the modern theory of wave motion and critical layers in rotating and/or stratified flows. [Pedley 2001, Stuart 1986]

Although these narrow regions occupy a relatively small proportion of the planet's fluid environments, they have to be understood and modelled accurately because of their disproportionately large influence on the flows and related processes over large distances of order L, both in the atmosphere, oceans, rivers, cryosphere, and in the Earth's interior in flows through cracks in the rocks and in the liquid core.

There are three main ways in which these layers influence the regions adjacent to them. Firstly they act as 'transport barriers' when waves and or eddies on one side of a layer within a flow are blocked (or 'shielded'); energy can 'pile up' on one side or in the interior of the layer, but does not penetrate across it. The barrier effect can also lead to the absorption of the energy of incident waves and eddies leading, for example, to its transformations into amplified fluctuations, or generation of a mean flow within the layer. In both cases there may be intense local dissipation. Secondly, these layers can act as 'transport through ways' with significant fluxes of mass, momentum, or scalars that may be as large or larger than those in the adjacent flow or medium. For example, flows along cracks determine water and oil movement in many rock formations. Both of these types of layers can also form along the fluid–solid boundaries of the flow region, where similar wave/eddy dynamics can occur. The boundary inevitably reduces, but does not completely prevent, the meandering of the layers in fluid media across large distances, as the Gulf Stream demonstrates so vividly. The third type is where the layer is an isolated line singularity such as a vortex (e.g., a hurricane) or a line sink or a line plume (e.g., a volcano). Such a layer also influences the flow over a very large horizontal distance L_H much larger than the vertical height L of the central core of the layer.

The mathematical and computational analysis of the dynamics of these regions first has to focus on the sharp variations in length scales. Asymptotic analysis shows how over the scale L of the outer flow these regions are two-dimensional, one-dimensional or point-wise singularities in terms of the equations governing the outer-scale flow. But the interactions between them and the thin singular regions (where different terms in the governing equations are dominant) requires an 'inner' layer asymptotic analysis. In some situations this leads to a significant feed back on the outer flow, especially through the effects on the eddy/wave perturbations propagating from the inner region. Multiple time scale analysis shows how over extended periods of many perturbations the mean flow is affected – a key mechanism for the transfer of momentum between different layers of the atmosphere and ocean.

A quantitative measure of the significance of the thin layers on the scale L is that their contribution to the energy spectrum of large scale atmospheric/oceanic motions results from the line surface and point singularities associated with these regions. [Farge et al. 1990]

The accuracy of finite difference computational models depends on a mesh size d_{mesh} in relation to thickness of the layers d . If d_{mesh} is larger than d then only the outer scales of motion are resolved, and the internal structures of the layers are not explicitly calculated: their effects have to be introduced into the computation, for example, as a prescribed line/surface discontinuity or a point-wise singularity. With more highly resolved computations, the interactions between the complex unsteady features of the internal structure of the layers and the outer flows are quantified in detail, and theoretical concepts can be tested. As this workshop concluded, better modelling of the dynamics and processes in these zones contributes to the improved numerical modelling of all aspects of geophysical flows.

Moving from these mathematical generalities to examples in the atmosphere and the ocean, it is important to realize how the dynamics of the layers depend on their physical scale. On the synoptic scale the dynamics are determined by the mean inertial forces of the flow, together with the variations on the global scale of buoyancy and Coriolis forces. But because the flows in these regions are unsteady, large scale eddy motions also significantly affect the mean dynamics. The atmospheric jet stream from West to East in the upper troposphere and the oceanic Gulf Stream in the Atlantic (or Kuroshio in the West Pacific) are familiar examples. In these regions, whose locations fluctuate over 1000s of kilometers, the velocity is a maximum (cf type (ii) profile of figure 1). Other important atmospheric examples of type (ii) where there are sharp changes of the flow variables across the fluctuating layers are the Inter-tropical convergence zone near the equator and the polar vortex, the latter being notorious for its effect on concentrating stratospheric ozone in polar regions. Note that synoptic scale motions in the ocean and the formation of layers differ from those in the atmosphere because the flows are obstructed by the continents and because the most intense

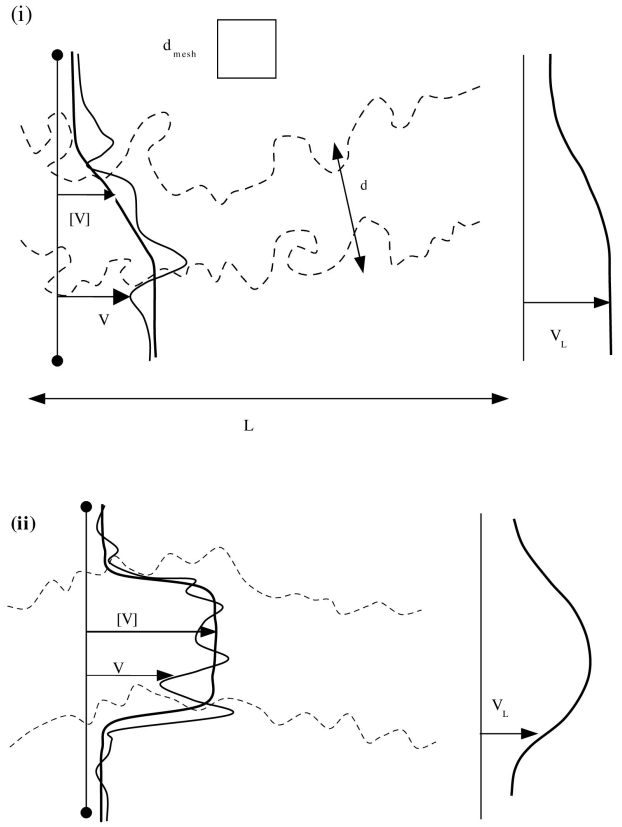

Figure 1 Schematic diagram of a thin layer (located within the dashed lines) in a geophysical flow, showing the length scales d and L, and typical profiles of some variable V for types (i) and (ii). Profiles are also shown for V conditionally averaged within (along) the layer, V_L, and averaged over a longer time scale, [V] . Many papers at the conference were concerned with the effect of varying the typical mesh size d_{mesh} of computations in relation to d.

vertical convection (and swirling eddies) occurs in the polar regions rather than the equatorial regions as in the atmosphere. Intense ocean boundary currents hundreds of kilometers wide form along continental coasts, driven by zonal winds – as mentioned above. Another special feature of the oceans is the generation of multiple zonal jets traveling East-West in the deep ocean, comparable to those visible in Jupiter's atmosphere. Their width (of the order of 100 – 1000 kilometers) is defined by the planetary Rhines scale $d_p = (u / f)^{1/2}$ where $f' = df / dy$ is the gradient of the Coriolis parameter f . [Galperin et al. 2006, Rhines 1979]

Over smaller 'mesoscale' distances there are many types of sloping layers or 'fronts' that separate regions of cooler or warmer air or water traveling with different velocities. In the atmosphere and the oceans these fronts have horizontal scales of 100 kilometers or 10 kilometers respectively, given by the internal Rossby deformation radius $L_R = Nh/f$, where h is the layer depth and N is the buoyancy frequency. The thicknesses of the layers in the vertical direction are less than about one kilometer or 100 meters respectively. The locations of these mesoscale layers are determined by variations of the large scale flows and also by changes in the boundary conditions. [Gill 1982] Elongated horizontal 'jets' of wind or ocean currents are driven by stresses induced by local variations of fluxes at the land surface or ocean surface, e.g., at coastlines. [Hunt et al 2004] Wind jets also affect the movement of sea ice and ocean upwelling along coast lines: an example of the significance of the critical layers for the interaction between the atmosphere and the ocean. Synoptic scale rotating storms or ocean circulations often tend to be initiated as mesoscale eddies, and then through mutual interactions grow into much larger structures.

Even thinner characteristic layers occur in horizontal stratified turbulent shear layers where vertical thickness d is broadly determined by a balance between the kinetic energy ρw^2 of the vertical velocity w and the potential energy of eddies $\rho N^2 d^2$ in the stable stratification, where N is the buoyancy frequency and ρ is the density. As in the large scale layers already described this length scale defines a transition between inertial and wave-like motions, i.e., $d = w/N$. These layers act as lids at the top of the atmospheric boundary layer and troposphere (where d ranges from 100 meters to 1 kilometer), or at the base of the ocean mixed layer about 100 meters below the surface where they separate the turbulence near the surface from the deeper stable quiescent regions dominated by internal wave motion (typically $d \approx 10$ meters). Such layers also occur intermittently within stable regions, as a result of local mixing, for example, at the tops of the clouds and where internal waves from mountains or convective regions are blocked and transform into turbulence at a critical height. Because most global scale atmosphere/ocean computational models cannot calculate explicitly the turbulent eddies and waves generated at comparable scales, formulaic models are in practice based on detailed research studies of these layers (see below). Layers of clear air turbulence in the stratosphere continue to occur unpredictably.

The thin layer flows that are predicted to occur in the liquid core of the earth also have distinct characteristics at the synoptic, mesoscale and turbulence layer scales; firstly because the space occupied by the liquid zone lies between a solid at the central sphere of the planet and the outer mantle. Because of the slow velocities, Coriolis forces are relatively stronger than in the ocean and atmospheric flows with the result that 'Taylor column' layers could form parallel to the Earth's axis of rotation, extending from the edges of the central core to the outer mantle. [Hide and Stewartson 1972] The second qualitative difference lies in the nature of the thermal convection; because of the slow diffusion of heat from the hot core, the convection in the liquid core is in the form of unsteady 'puffs', unlike the narrow and long lived thermal plumes driven by the faster radiatively controlled heat transfer processes at the Earth's surface. [e.g. Hunt et al. 2003]

Developments in adaptive grid methods and their use

N. Nikiforakis and K. Bates (Cambridge) reviewed recent developments in the formulation and application of adaptive, variable resolution grids to the computation of geophysical flows. In aeronautical computations these adaptive grids have been used for over twenty years to resolve fine details of flow around shock waves. But to resolve the highly fluctuating critical regions of atmospheric and oceanic flows such as fronts, fine scale grids are needed that move and adjust corresponding to the locations as well as the dynamics and kinematics of these layers and other singular regions (such as hurricanes or ocean eddies). The advantage of this approach is that in the external region, where flow gradients are very weak, the grids can be relatively coarse so that the total number of meshes is much less than if fine scale meshes are used everywhere. There are still some numerical problems to overcome before these methods can be used regularly in operational weather forecasts. For example, wave motions can be artificially trapped in areas where the grid size varies very rapidly. It is estimated that such methods may be 100 times faster than fixed grids for the same computational capacity and the same level of accuracy. For example, Nikiforakis has demonstrated how frontal dynamics are well represented in global atmospheric models run on the fastest and the largest capacity computer system in the world: the Earth Simulator in Japan.

The adaptive methods also reduce the serious errors that tend to occur with current models when computing the air flows over mountains with steep gradients. Such errors may greatly affect the global circulation, as well as local precipitation.

C. Pain (Imperial) described the Imperial College computational model for ocean dynamics, which is based on highly anisotropic mesh adaptation. Mostly the mesh is constant in time, but there is some adjustment for calculating particular effects. The results showed complex unsteady behaviour that occurs in the critical layers above ocean currents traveling down steep slopes, and also in swirling ocean eddies driven by surface cooling. These adaptative methods are particularly suitable for the critical regions where the boundary of the flow domain is rapidly deforming at length scales that are small compared with the land masses, such as where the ocean depth decreases to zero at the coast or on sand banks etc. The models describe the distortions to ocean currents and waves, especially when the sea level falls and rises with the tides and the interactions between the sea and the solid bottom lead to the latter drying out or saturating. The current concern about the biological degradation of the oceans is a vitally important application of accurate ocean modelling, because biological processes tend to be focused within sharply defined regions of the ocean, e.g. where fronts cause upwelling of nutrients.

A. Mahalov, A. Moustaoui, B. Nicolaenko (Arizona State University) presented the results of some of the highest resolution computations ever performed of atmospheric jet streams. These are stably stratified turbulent layers with thickness of about 1 kilometer located at the top of the troposphere or in the lower stratosphere. Pseudo-spectral discretisation is used in the horizontal with an adaptive (but fixed) grid in the vertical with grid spacing down to a meter in the thin layers at the top and bottom of the jet. The results showed how the turbulent eddies on these interfaces generate waves that propagate energy vertically out of the jet, as predicted theoretically but not previously seen in computations. These are similar to the waves produced by atmospheric or ocean flows over mountains or over sea mounts, as other computations have confirmed. For operational calculations, using an advanced mesoscale model (where the grid size is 5m), it was shown that similar wave–eddy–mountain interactions are indeed simulated, but their peak levels are under-predicted because of the coarser resolution.

J. Chan (Polytechnic University of Hong Kong) gave a clear example of the use of a high resolution adaptive computation to develop new modelling concepts for a complex flow over non-uniform conditions at the Earth's surface. He computed the flows surrounding a tropical cyclone (or hurricane or typhoon) as it approaches a coast line and then weakens as a result of the sudden increase in surface resistance and cessation of water vapour flux. He used a mesoscale model over a large area (6750 kilometers by 6750 kilometers horizontally), but 'triply-nested' with three scales of grid spacing varying from 45 kilometers to 5 kilometers near the cyclone centre. The computations showed how coastal interaction distorted the vorticity distribution in the cyclone, causing it to move towards the coast. This effect is not included in most current weather prediction models. Also the associated up-drafts produce localized patterns of intense rain and often flooding, which could not be explained without this detailed study.

Advances in aspects of climate models

W. Norton (Reading) presented one of the most advanced types of the traditional fixed grid model that is used by global modelling groups in UK universities. The horizontal scales of the mesh sizes are about 100 km in the atmosphere and 33 kilometers in the ocean, with 80 mesh levels in the vertical. This has enabled more critical regions to be included and also more types of phenomena to be described, such as Easterly waves of clouds in the tropics and intense localized areas of precipitation. But other critical regions and layers are not resolved, which can have a major effect on prediction of local climate, e.g., along coastlines (where most of the world's population is concentrated!) He pointed out that local models are needed for such regions.

Improving the models and methods for computation of the growth, movement, and breaking-up of the sea ice, especially at its outer edge, is an essential part of the general programme of increasing the accuracy of global climate models. This is because the sea ice processes help drive the ocean currents in polar regions and beyond (the thermo-haline circulation), and as the ice area changes (which affect 10% of the Earth's surface) the temperature is predicted to rise in these regions by as much as $8-10°$K over the next hundred years!

D. Feltham (UCL and British Antarctic Survey) reviewed developments in modelling the ice deformation and the interactions between ice and the sea beneath it. These extend the current Hibler model to allow for the anisotropy of the ice and its characteristic deformation in thin strips where impinging floes form ridges. The air-sea processes in the narrow cracks between separating ice floes tend to dominate the heat transport between the ocean and the atmosphere. As in meteorological modelling, approximations are required to represent the effects of multiple elements or layers in a numerical model where the grid size is larger than that of the physical elements (e.g., clouds or ice floes). By representing one large 'typical' element in each grid box some of the detailed physics is modeled, but by omitting to model the distribution of sizes of the elements and their interactions some significant errors may still arise.

J. Ridley and H. Banks (Met Office) described their fixed grid model (on a scale of about hundred kilometers) for sea ice processes in the Hadley Centre climate model. A 'tiled' approach is used in which air-sea and air-ice fluxes are partitioned within each grid box. As is increasingly common in meteorological numerical modelling, this code is constructed so that components can be exchanged with codes of other research groups. From the variability of multiple computations for a particular geophysical situation using different codes and different elements, errors in sea ice predictions can be estimated. But it is also recognized that all such models tend to be based on certain common assumptions and therefore could lead to systematic errors in all models.

A. Orr (European Centre for Medium-range Weather Forecasts) described a detailed study of strong down-slope winds predicted over the Italian Alps, using global atmospheric models (with 25 kilometers horizontal resolution and 91 vertical levels). He also showed how the effect of the upper level waves on an inversion layer could amplify the flow perturbations at ground level, even causing reversal of the flow. In February 2006, this effect strongly influenced the forecast for the opening of the Torino winter Olympics! Higher resolution computation with a mesoscale model (by the Italian National Meteorological Service) was necessary to provide more accuracy in such a situation. Flow over the Rocky Mountains has similar requirements (see below).

Two reviews of current trends in developing and interpreting global circulation models (GCMs) brought out the central importance of improving the modelling and computation of critical regions. M. Collins (Met Office) first noted that the rate at which the grid size of GCMs can decrease is tied to the rate at which computers are increasing in speed – a doubling is possible approximately every six years according to Moore's Law. He estimated that in GCMs the sizes of fixed grids would only reach ten kilometers (from the present hundred kilometers) in about twenty to thirty years. However, some of the extra computer capacity is being used to increase the complexity of the modelling, e.g., allowing for the effects of aerosols in clouds or including feedbacks from biological processes. To understand better the consequences for the reliability of GCMs of the considerable uncertainty associated with the parameters in these model components, Collins described studies in which twenty nine of the parameters were varied over physically plausible ranges of values. It was found that this could lead to an uncertainty in the rise in global temperature, under a representative scenario of a doubling of CO_2, of about 3°K. However most of the variability arose from the uncertainty about the complex edge turbulence/wave entertainment processes at the tops of clouds and in inversion layers (which indeed previous research had indicated; Fernando and Hunt 1996). This finding suggests where more research is still needed.

M. Davey (Met Office and UCL) explained how global circulation models are now also being used to predict the average weather condition over periods ranging from a few months to decades ahead. Although some 'seasonal' predictions are still based on long term statistical relations, increasingly the dynamical models are supplementing and even replacing the statistical approach. The critical feature of both types of prediction is that they depend sensitively on the variations of the temperature of the ocean surface layer, and the depth of the ocean thermocline that lies 50–300 meters below the surface. The dynamics of this thin layer are critical since the fluctuations in its structure and depth may be the main determining effect on natural climate variability for timescales of seasons to decades. How this effect interacts with a general rise in global temperature is one of the issues being considered in international and EU projects to compare the results from various climate models, as well as the results of varying the assumptions that go into them. At the same time further developments in modelling are being introduced, such as those concerned with the dynamics of critical regions such as the 'upwelling' near continental shelves.

Further theoretical and computational developments

Fundamental developments in grid based computational systems for fluid flows have led to better representation of the time dependence of variables as they are carried (or 'advected') by the flow at arbitrary angles relative to the grid. The limitations and advantages of a semi-Lagrangian method were analysed by M. Zerroukat (Met Office). He pointed out that certain properties of the flow are not conserved that should be (e.g., the concentration of a non-reactive gas in the atmosphere). This may also be true for certain adaptive schemes. To avoid continually having to introduce ad hoc corrections in such numerical schemes (as is done at present), a new approach was proposed that when tested on the critical layers surrounding the polar vortex conserved advected gaseous components more accurately, but without any loss in computational speed.

E. Johnson (UCL) analyzed thin stable layers flowing over three dimensional mountains or sea mounts using the theory of shallow water flows, in a rotating frame. When the Froude number F is closed to the critical value $F=1$, non-linear Schrödinger-like solutions are needed to describe the sub- and super-critical wave patterns that form around the mountain/sea mounts on the stable layers or sea surface, with and without rotational effects. Results from linear theory, which form the basis of most estimates for the effects of the drag of mountains in weather forecasting models for example, are broadly correct except when $F \approx 1$. Such situations exemplify how local flows may have a large effect on meteorology over much wider areas, even up to a continental scale.

S. Dash (Indian Institute of Technology) analyzed the relations between variations in the snowfall on the Himalayas, the oceans and the climate in India over the past century. He argued that the meteorology and the form of the stratified flow over the Himalayas is being affected by climate change (as it is over other high mountain ranges), which may be associated with a weakening South West monsoon. There is quite a delicate dynamical balance involved, which the current global scale numerical models cannot accurately describe, for which mesoscale models are beginning to be used to greater effect.

Narrow layers of intense winds are another pronounced feature of flow over mountains; R. Kerr (Warwick) analysed the vorticity dynamics and local scale computations of an extreme example of such winds over the Rocky Mountains, when the down slope mean flows driven by local buoyancy forces and high intensity turbulence were amplified by the effect of high amplitude waves above the mountains. He concluded that only with high resolution computations, with grids of about a hundred meters, can the intensity of such phenomena be predicted – which is particularly necessary for aircraft operation. Kerr also pointed out how mountain effects, storms and other major perturbations to the atmospheric flows on meso and synoptic scales can grow in scale as they interact with all the other horizontal motions on these scales: another reason why higher resolution modelling can lead to qualitative changes in predictions.

In a broader theoretical analysis of waves, vortices, fronts, and larger scale phenomena in ocean–atmosphere dynamics, J. Norbury (Oxford) compared the essential properties of the two main approximate models that have been proposed: quasi-geostrophic and semi-geostrophic models. [Norbury and Roulstone 2002] They are based on the assumptions that the phenomena occur over lengths larger than the mesoscale where inertial forces generally have a smaller effect than buoyancy and Coriolis forces; but in certain thin layer regions, such as unsteady fronts, they may have a controlling effect. The analysis of the different fast and slow modes of the geophysical equations of stratified rotating flow indicates which aspects of the flow approximate numerical solutions can predict accurately, and how the aspects that they are neglecting might affect the predictions.

At the edges of regions with strong turbulence, such as the top or bottom of a boundary layer, the distributions of the flow variables tend to have sharp gradients, with a thickness that is much less than the length scales of the adjacent turbulence. At the same time the interfaces fluctuate with cusp-like indentations (or ‘engulfments’). I. Eames (UCL) developed a mathematical model to explain how the gradients of turbulence intensity determine the two and three dimensional dynamics of these sharp interfaces, and thence why the small scale ‘nibbling’ instabilities over the whole interface drive the outward movement of the turbulent region more than the engulfment that occurs intermittently, but on a larger scale. He drew attention to the geophysical applications of these turbulent and entrainment processes, the most dramatic being the high gradient of turbulence in the sharply defined rotating ‘eye-wall’ of hurricanes.

E. Shuckburgh (Cambridge) pointed out how barriers to the transport of scalars also occur in regular flow patterns, e.g., by a collection of vortices, if the stream lines and particle paths do not diverge chaotically (except in narrow regions). But where the gradients of the vorticity build up, associated with vortex motion in thin layers, the barriers become more like those found at the edge of turbulent layers. She pointed out how different physical explanations and different terminology have been proposed to account for barrier effects. On the global scale, the magnitude of the gradient of potential vorticity is the usual criterion for significant dynamical barrier effect – as is observed in atmospheric jets and the edge of the polar vortex. In many cases this is equivalent to the mechanism of the vertical ‘sheltering’ of eddy motion by critical layers. As in other such regions, high levels of fluctuations can occur in these layers, even on this scale. Significant transport of energy and momentum can occur, and these layers can act as wave-guides for synoptic scale Rossby waves.

N. Wedi (European Centre for Medium-range Weather Forecasts) focused on an unusual synoptic scale critical layer associated with long term oscillations that are not forced externally (as in the inter-tropical convergence zone or the polar vortex edge) but instead result from an instability probably resulting from internal wave motion. This is the 28-36 month Quasi-Biennial Oscillation (QBO) that is formed in the mid to lower stratosphere at 20–40 kilometers altitude and propagates downward at about 1 kilometer per month. Not only are the length scales of the wave/eddy motion in the QBO region too small to be simulated by operational weather/climate models, but identifying the relevant dynamical processes is also uncertain. However, Wedi noted that the laboratory experiment of rotating flows with superimposed waves which could simulate some aspects of the QBO (Plumb and McEwan, 1978) could now be described by very detailed computational models. In particular, these confirm that slow period internal waves could interact to form critical layers that persist over much larger periods. [Galmiche and Hunt, 2003]

Conclusion

This conference highlighted the need for specialists working on the numerical and mathematical modelling aspects to exchange ideas about the different approaches that are contributing to the undoubted progress being made in the prediction and theoretical understanding of critical regions in geophysical flows. The conference would have benefited from more contributions from experts working on the large scale flows, in order to relate the local to wider dynamics. Perhaps such a meeting could be held before long!

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