

On the Nusselt number for frazil ice growth—a correction to “Frazil evolution in channels” by Lars Hammar and Hung-Tao Shen

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ABSTRACT

The growth (melt) rate of frazil ice is governed by heat transfer away from (towards) the ice crystal, which can be represented by the Nusselt number. We discuss choices for the Nusselt number and turbulent length scale appropriate for frazil ice and note an inaccuracy in the study “Frazil evolution in channels” by Lars Hammar and Hung-Tao Shen, which has also led to potentially significant errors in several other papers. We correct this error and suggest an appropriate strategy for determining the Nusselt number applicable to frazil ice growth and melting.

RÉSUMÉ

Keywords: Frazil ice, Nusselt number, heat transfer, turbulence, boundary layer.

1 Introduction

In recent years, mathematical models of frazil ice formation have been employed in studies of sea ice (Smedsrud and Jenkins, 2004; Svensson and Omstedt, 1994) and lake and river ice (Hammar and Shen, 1995; Wang and Doering, 2005). Following the extensive reviews of Daly (1984, 1994), all of these studies consider the growth (melt) rate of frazil ice to be limited by the transfer of heat away from (towards) the frazil crystal. Considerations of salt diffusion also apply in seawater (Smedsrud and Jenkins, 2004), but these are beyond the scope of the present discussion.

The Nusselt number represents the ratio between the actual (potentially turbulent) heat transfer rate and the heat transfer rate through conduction alone. If the turbulent heat transfer rate is given by $q\Delta T$ and the conductive heat transfer rate by $k_w\Delta T/l$, then the Nusselt number may be written

$$\text{Nu} = \frac{ql}{k_w}. \quad (1)$$

In this expression, q is the heat transfer coefficient, k_w is the thermal conductivity, and l and ΔT are characteristic length and temperature scales; l is the thermal boundary layer thickness over which the temperature changes by ΔT . Since frazil ice only forms in turbulent water, Nu necessarily appears in the growth and melting terms of all frazil ice models and its choice is thus an integral part of frazil ice modelling.

When a motionless frazil crystal is suspended in a stagnant fluid, $\text{Nu} = 1$, implying that heat transfer from the crystal occurs by conduction alone. In the more usual case where the crystal has a velocity relative to the fluid, the motion may increase the heat transfer rate. If the crystal is smaller than the smallest existing eddies, it feels the fluid motion as varying linearly with position and Nu increases a little above 1 as its average relative velocity increases and convection of heat grows in importance. When a crystal is larger than existing eddies, they will enhance heat transfer by mixing the thermal boundary layer next to the crystal surface; this can lead to a value of Nu that is considerably greater

than 1. The Kolmogorov length scale

$$\eta = \left(\frac{\nu^3}{\epsilon} \right)^{1/4} \quad (2)$$

(where ν is the viscosity of the fluid and ϵ is the turbulent dissipation rate) gives the scale at which viscous forces in the fluid dissipate the energy cascading down from turbulent eddies at larger scales, and therefore is the scale of the smallest eddy that is sustainable in steady state. We can thus use η to decide when l is larger than existing eddies and the boundary-layer heat transfer is affected by turbulence. If we assume a viscosity of $\nu = 1 - 2 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ and a dissipation rate of $\epsilon = 0.1 - 4 \times 10^{-6} \text{ m}^2 \text{ s}^{-3}$ (choices encompassing the range used by the studies referred to in the references of this paper) then we find that η takes values of 0.06 – 1 mm.

2 The characteristic length scale of the thermal boundary layer

Frazil ice crystals are shaped like discs and grow preferentially along the disc edge rather than the faces. Previous authors have chosen a variety of quantities to use for the characteristic length scale l . Most authors use the crystal disc radius r (Daly, 1984, 1994; Hammar and Shen, 1995; Holland and Feltham, 2005; Smedsrud, 2002, 2003; Smedsrud and Jenkins, 2004; Wang and Doering, 2005), but the length $\sqrt{3/8} r$ (based on the crystal's total surface area) (Ye and Doering, 2004), the disc thickness (Svensson and Omstedt, 1994, 1998) and half of the disc thickness (Bombosch and Jenkins, 1995; Jenkins and Bombosch, 1995; Khazender and Jenkins, 2003) have also been used.

Since maximum frazil disc radii are of the order of 1 mm while maximum disc thicknesses are of the order of 0.1 mm (Forest, 1994), the choice of which quantity to use for l is crucial in determining whether $l > \eta$ and boundary-layer mixing is important. Another important requirement is that the choice for l should match the choice of length scale over which the relevant temperature gradient in the growth term is taken. This is not strictly the case in some published works, which assume r for the latter and then use a constant value of Nu without testing against the appropriate η (Holland and Feltham, 2005; Smedsrud, 2002, 2003; Smedsrud and Jenkins, 2004).

Wadia (1974) calculated that more than 80% of a suspended particle's relative velocity was accounted for by eddies on scales larger than r (Daly, 1984). Therefore, we argue that r is the scale beneath which eddies mix the frazil crystal's boundary layer rather than moving the crystal; whether any such eddies exist is established by comparing r to η . Using any of the smaller lengths mentioned above will overestimate the influence of these relatively small eddies on the particle's relative velocity and thus underestimate their effect on mixing of its thermal boundary layer.

3 The Nusselt number

Irrespective of the choice of l , one must decide upon a formulation to use for Nu in the event that $l > \eta$ and the boundary layer

heat transfer is affected by turbulence. To simplify the following discussion, we choose $l = r$ and define the quantity $m^* = r/\eta$ to be the ratio between the disc radius and the Kolmogorov length scale. Daly (1984) developed a parameterisation of Nu for different values of m^* from experimental and theoretical studies of suspended particles. This parameterisation is indirectly presented in terms of the ‘‘turbulent’’ Nusselt number

$$\text{Nu}_T = \frac{q\eta}{k_w}, \quad (3)$$

which is the value of Nu referenced to the Kolmogorov length scale. This is potentially confusing because at this scale Nu should theoretically be only weakly affected by turbulence as there are no sustainable eddies smaller than the crystal. Nu_T does, at least, reveal the relative heat transfer coefficient for each disc radius at a common length scale.

Hammar and Shen (1995) were the first authors to employ the parameterisation of Daly (1984) in a refereed journal article, but unfortunately they seem to have adopted the formulation for Nu_T in their frazil growth rate rather than Nu. This has been copied by recent studies (Wang and Doering, 2005; Ye and Doering, 2004) and assumed correct in the conclusions of Smedsrud (2002). The invalidity of this formulation is demonstrated by the fact that (as stated in the articles) Nu_T increases as the crystal size decreases, whereas turbulent mixing of the thermal boundary layer should decrease once $m^* \leq 1$ as no sustainable eddies are smaller than the crystal radius. It can be easily deduced that $\text{Nu} = m^* \text{Nu}_T$, so if the smallest discs (which are usually worst affected) have radius 0.004 mm [Hammar and Shen, 1995; Wang and Doering, 2005; Ye and Doering, 2004], the growth rate for those crystals is at worst $m^{*-1} \approx 95$ times too large (when $\eta = 0.381$ mm based on $\nu = 2 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ and $\epsilon = 3.8 \times 10^{-4} \text{ m}^2 \text{ s}^{-3}$ (Hammar and Shen, 1995)) and at best $m^{*-1} \approx 14$ times too large (when $\eta = 0.0574$ mm based on $\nu = 10^{-6} \text{ m}^2 \text{ s}^{-1}$ and $\epsilon = 9.2 \times 10^{-2} \text{ m}^2 \text{ s}^{-3}$ (Ye and Doering, 2004)).

4 Conclusions and correct formulation

We have argued that, in order to avoid underestimating the effects of turbulence on heat transfer from a frazil disc, one should use the disc radius as the length scale of its thermal boundary layer. We have also shown that an inaccuracy in Hammar and Shen (1995) can cause errors in the growth rate of modelled frazil ice. To remedy these problems, we recommend that future authors use r as the thermal boundary layer length scale and the following formulation for Nu, as originally intended by Daly (1984). For $m^* \leq \text{Pr}^{-1/2}$, where $\text{Pr} = \nu/\kappa$ is the Prandtl number (κ is the thermal diffusivity), heat transfer is dominated by diffusion, so

$$\text{Nu} = 1 + 0.17 m^* \text{Pr}^{1/2}. \quad (4)$$

For $\text{Pr}^{-1/2} < m^* \lesssim 10$ the velocity distribution is still approximately linear (Batchelor, 1980), but heat transfer is influenced more by convection. We use the following

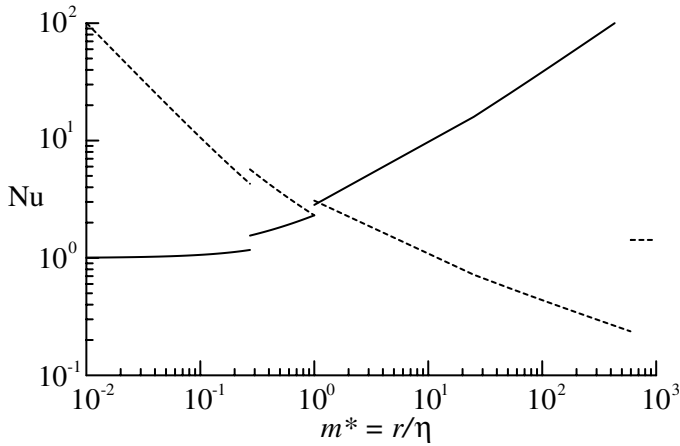


Figure 1 Values of Nu over the range of m^* for both the inaccurate formulation adopted by Hammar and Shen (1995) (dashed line) and the new formulation (solid line). Both curves are plotted for $Pr = 13.44$ and $\alpha_T = 0.2$.

for $Pr^{-1/2} < m^* \leq 1$:

$$Nu = 1 + 0.55 m^{*2/3} Pr^{1/3}. \quad (5)$$

For $m^* > 1$ heat transfer in the boundary layer is governed by turbulent mixing. If $\alpha_T m^{*4/3} \leq 1000$, where α_T is the turbulence intensity of the fluid (Daly, 1984), heat transfer is in the low-turbulence regime

$$Nu = 1.1 + 0.77 \alpha_T^{0.035} m^{*2/3} Pr^{1/3}, \quad (6)$$

and for $\alpha_T m^{*4/3} > 1000$, heat transfer is in the high-turbulence regime

$$Nu = 1.1 + 0.77 \alpha_T^{0.25} m^* Pr^{1/3}. \quad (7)$$

Finally, we note that some of the constants in Eqs. (6) and (7), which agree with Daly (1984), differ from those of Hammar and Shen (1995); we are unsure as to why this is the case. The values obtained from applying both these formulae and the incorrect versions of Hammar and Shen (1995) are shown in Fig. 1. As can be seen from Fig. 1, use of the inaccurate formulation adopted by Hammar and Shen (1995) leads to error in the Nusselt number, and therefore the growth rate of frazil crystals, by a factor of up to about 500, but more typically of order 10 – 100.

Notation

- k_w = Thermal conductivity ($W m^{-1} K^{-1}$)
- l = Turbulence length scale appropriate for frazil disc (m)
- m^* = Ratio between frazil disc radius and Kolmogorov length scale
- Nu = Nusselt number
- Nu_T = “Turbulent” Nusselt number
- Pr = Prandtl number
- q = Heat transfer coefficient ($W m^{-2} K^{-1}$)
- r = Frazil disc radius (m)
- α_T = Turbulence intensity of the fluid
- ϵ = Turbulent dissipation rate ($m^2 s^{-3}$)

- η = Kolmogorov length scale (m)
- ΔT = Temperature difference (K)
- κ = Thermal diffusivity ($m^2 s^{-1}$)
- ν = Viscosity ($m^2 s^{-1}$)

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