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METHOD OF DETERMINING A GEOPHYSICAL-SCALE SEA ICE RHEOLOGY FROM LABORATORY EXPERIMENTS

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ABSTRACT

We present a methodology that allows a sea ice rheology, suitable for use in a General Circulation Model (GCM), to be determined from laboratory and tank experiments on sea ice when combined with a kinematic model of deformation. The laboratory experiments determine a material rheology for sea ice, and would investigate a non-linear friction law of the form $\tau \propto \sigma_n^{2/3}$, instead of the more familiar Amonton's law, $\tau = \mu\sigma_n$ (τ is the shear stress, μ is the coefficient of friction and σ_n is the normal stress). The modelling approach considers a representative region R containing ice floes (or floe aggregates), separated by flaws. The deformation of R is imposed and the motion of the floes determined using a kinematic model, which will be motivated from SAR observations. Deformation of the flaws is inferred from the floe motion and stress determined from the material rheology. The stress over R is then determined from the area-weighted contribution from flaws and floes.

SEA ICE DYNAMICS

Sea ice exerts a strong influence on climate due to the high albedo of the sea ice cover, by insulating the ocean from the atmosphere, and through its influence on global ocean circulation. The atmosphere and ocean exert forces on the ice cover transporting ice over large distances and shifting the ice margin. Changes in sea ice cover are one of the largest uncertainties in predicting global temperature increase (Carson, 1999).

The forces affecting sea ice dynamics are summarised in the momentum equation used in all modern sea ice models (Hibler, 1979) but on timescales of climatic interest, the force balance is typically dominated by atmospheric drag, oceanic drag and ice interaction force, which arises from ice floes rubbing against each other or over-riding each other to form pressure ridges, with the precise balance between these forces depending upon time and location (Steele et al., 1997).

Tuning of model parameters such as air to ocean drag ratios and ice strength is

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employed in order to achieve qualified success in reproducing data sets of sea ice extent, concentration and buoy motion (e.g., Harder and Fischer, 1999). This tuning is required partly because current sea ice models do not resolve physical processes below grid sizes of 100 km (e.g., Coon et al., 1998). We suggest that such tuning can, at best, provide a good fit to reality over limited periods of time. This is supported, for example, by calculations with different ice strengths (Steele et al., 1997). Sustained accuracy over decadal and longer timescales requires models with parameters that can be independently assessed. Here, we focus on the ice interaction force i.e. sea ice rheology.

DEVELOPMENT OF A SEA ICE RHEOLOGY WITH NO TUNING

In order to determine a sea ice rheology with parameters that can be independently assessed, we must relate sea ice rheology on climatically important length scales to material properties of sea ice as measured in the laboratory. Our methodology is:
(a) Perform a suite of laboratory experiments on saline ice to measure sea ice friction;
(b) Validate these experiments at an intermediate scale in a model ice basin;
(c) Use these experimental results to generate a sea ice constitutive relation suitable for inclusion into GCMs and investigate sensitivity to kinematic assumptions using satellite observations of the polar pack.

EXPERIMENTAL WORK

Sea ice deformation creates thick ice through pressure ridging and thin ice through creation of open-water leads. Thus, it is essential to incorporate brittle-discontinuous processes into rheological models, e.g. Hibler and Schulson (2000). We argue that *tensile fracture* and *frictional sliding (shear rupture)* in shear govern brittle processes in sea ice dynamics. Note this is rather different from the view expressed by Hibler and Schulson (1997), which gives a leading role to shear fracture. We believe that the stresses needed for shear fracture in ice are too high for it to be important on a geophysical scale (Sammonds et al., 1998). We argue that the ice cover is broken under tension (rather than shear fracture), and stresses arise during shear deformations by frictional sliding (shear rupture) at the interface between floes. During convergence with pressure ridge building, the compressive strength is determined by the frictional sliding between the blocks that make up the ridge and the forces acting against buoyancy that create gravitational potential energy, see Rothrock (1975). If the ice cover is simultaneously shearing, the strength is also determined by friction losses from floes sliding past each other, Pritchard (1981).

Most friction experiments on ice, using a slider block, have been done at high temperatures, low normal stresses, and sliding rates where frictional heating leads to the formation of a lubricating water film, e.g., Jones et al. (1991). Results are modelled by Amonton's classical friction law, $\tau = \mu\sigma_n$ and interpreted in terms of the lubricating effect of the water film. Thus stick-slip friction, recently observed in sea ice (Sammonds et al., 1998), has been overlooked, though this may play a role in sea ice dynamics and can explain behaviour in ice tank experiments (Tuhkuri, 1999).

Rist (1997) has re-analysed ice friction data over a range of normal stresses and temperatures, where surface melting is unlikely to occur, and has shown that ice follows a well-defined *non-linear* friction law in which $\tau \propto \sigma_n^{2/3}$. Rist attributes this behaviour to elastically deforming asperities undergoing shear fracture. Similar behaviour has been observed for sea ice on limited data (Sammonds et al., 1998). The implication is that the coefficient of friction for sea ice is actually *dependent* on normal stress, but *independent* of temperature, surface roughness and sliding rate. This is the complete converse of the traditional view of sea ice friction and suggests a fresh experimental study of sea ice friction is needed.

It has been shown that the critical slip displacement, D_C, is directly related to the critical rupture growth length, L_C, and also to the *fractal* characteristics of the sliding surface (Ohnaka and Shen, 1999). In order to derive a *scaling* relation for shear rupture it is therefore necessary to measure shear stress τ, slip displacement D, rupture growth length L and the fractal characteristics of the sliding surface.

Proposed laboratory work

(i) We propose measurements of shear stress against slip displacement, for a range of normal stresses that will span low stresses of slider block tests and higher stresses of the triaxial test for a range of temperatures and sliding rates. This will allow a constitutive relation for shear rupture, $\tau(\sigma_n, T, \dot{\varepsilon}, D) = f(\sigma_n)g(T)h(\dot{\varepsilon})i(D)$, where T is temperature and $\dot{\varepsilon}$ is strain rate, to be determined (i.e., the dependence of the friction coefficient μ). The range will also cover where frictional heating causes surface melting, though lubricating effects of sea water would be measured directly using a saline bath; (ii) For a limited number of experiments with a strain-gauged specimen, we propose measurements of the rupture growth length L along with shear stress and slip displacement, for a range of surfaces roughness. Surface roughness would be measured with a profilometer and fractal characteristics calculated (Sammonds and Ohnaka, 1998). This allows determination of the relation between D_C, L_C, surface fractal characteristics and their dependence on peak shear stress and stress drop, and hence a scaling relationship; (iii) In order to validate the constitutive relation for shear rupture in sea ice and scaling relations determined in the laboratory, we propose experiments at an intermediate scale in a model ice basin. Experiments would be performed on both model ice, a granular material with scaled down mechanical properties, and saline ice. The first set of experiments will study collisions of discrete ice floes and a floe and ice sheet, in order to validate the laboratory constitutive model for the ice-ice interface under both normal and tangential forcing. The second set of experiments will measure the propagating rupture growth length in the ice during frictional sliding, where two sheets raft and slide over each other.

CALCULATION OF A GEOPHYSICAL-SCALE RHEOLOGY

A yield envelope describing plastic flow for a frictional law would be obtained by plotting the measured critical stress state in principal coordinates of strain rate. The Mohr-Coulomb shear failure branches will be determined by the friction law. Tensile strength would be incorporated through the tensile opening fracture toughness (Dempsey et al., 1999). We expect that the envelope for the initiation of frictional slip would be non-linear and independent of temperature for the environmental conditions of the polar pack, in contrast to the linear failure envelope employed by Hibler and Schulson (1997). It would also be necessary to approximate the sub-failure mechanical response of the ice; previous workers have assumed an elastic response (e.g., Coon et al., 1974) and visco-plastic response (e.g. Schulson and Nickolayev, 1995). From this constitutive behaviour, a material, constitutive equation can be determined which includes information about the yield curve, plastic flow rule *and* sub-failure response.

Consider a region R of dimensions of 100 km × 100 km containing a collection of ice floes or floe aggregates (with size, say, of 1–10 km) separated by thin, rectilinear regions of thinner ice (flaws) and open water. We propose determining the geophysical-scale rheology of sea ice by imposing strain rates on the region R, and calculating the stress required to cause this deformation. We, therefore, need a description of the constituents of R (quantity, type and distribution of ice) and a *kinematic* model that relates the deformation of the constituents to the deformation of the region as a whole, e.g. Moritz and Ukita (2000). The choice of the composition of R and the deformation of its constituents would be investigated using data from the Radarsat Geophysical Processing System, see Moritz and Stern (2001). Similar, purely theoretical approaches have been pursued by Ukita and Moritz (1995), with square and diamond-shaped floes, and Thorndike (1987), Ukita and Moritz (2000) who considered an isotropic, random distribution of convex polygonal floes.

Since the flaws are weaker than the floes, these will preferentially fail so that the composite strain rate of R is (almost) entirely due to deformation of the flaws. As a first approximation, a suitable kinematic model would be simply to ascribe individual floe velocities according to $\mathbf{v}^k = \nabla\mathbf{u}\cdot\mathbf{x}^k$ where $\mathbf{x}^k$ is the position of the centre of the k^{th} floe and $\nabla\mathbf{u}$ is the velocity deformation tensor where $\mathbf{u}$ is the velocity appropriate to the continuum scale. This approach (Gray and Morland, 1994; Ukita and Moritz, 2000) allows easy calculation of the velocity difference and hence strain rate across each flaw separating pairs of floes. From the strain rate in a flaw, the material constitutive equation would be used to determine the flaw stress during divergence and shear. With convergence of a flaw, we would expect pressure ridge formation, and the convergent stress would be determined with the energetic arguments of Rothrock (1975) using the laboratory-determined, nonlinear friction law.

At each floe edge, continuity of normal traction must be applied across the floe-floe or floe-flaw interface. This allows, to leading order, the determination of the mean stress in each floe using Signorini's mean stress theorem (e.g., Gray and Morland, 1994) as the other forces in the floe (except gravity) play a secondary role. The geophysical, continuum-scale, mean stress field over the region R is calculated from the area-weighted sum of the local, mean floe and flaw stresses. Thus, by varying the imposed, continuum-scale velocity field $\mathbf{u}$, the continuum-scale constitutive behaviour can be determined. This is expected to be anisotropic (Hibler and Schulson, 2000). The explicit, continuum-scale constitutive equation calculated could form the rheology module used in sea ice dynamics models in GCMs or stand-alone models.

The choice of composition and kinematic model are important for calculating the geophysical-scale rheology. Numerical experiments would be performed, indicating the sensitivity of rheology to ice concentration, and thickness distribution in the flaws (such concerns have been highlighted by Coon et al. (1998)). In order to build intuition, it would be helpful to consider the limiting cases of one, two and several active leads in an (otherwise) isotropic field of sea ice (the case of one lead has been solved by Hibler and Schulson (1997)) and consider open water fractions. Modified kinematic models would also be investigated, e.g. stochastic fluctuations about the mean flow velocity $\nabla\mathbf{u}\cdot\mathbf{x}^k$ (following Thorndike, 1987), decomposition of strain applied to R into sliding along active planes (e.g., Tremblay and Mysak, 1997), and the possible role of stick-slip behaviour. Where possible, these models would be constrained with Radarsat data.

DISCUSSION

We have presented a methodology that allows a sea ice rheology, suitable for use in a GCM, to be determined from laboratory and tank experiments on sea ice when combined with a kinematic model of deformation. The validation of this rheology can only be obtained through comparison of detailed sea ice dynamics simulations with a range of data sets; stress measurements made during US field experiments such as SIMI and SHEBA will provide a partial test of predictions using the rheology.

We believe that sustained accuracy in simulations over decadal and longer timescales necessary for climate prediction require a sea ice rheology with parameters that can be independently assessed. Although the methodology described here does not eliminate the need to choose parameters, especially in the determination of a realistic kinematic model, it does allow a fairly explicit investigation of the role that sub-100 km scale physics, the material response of sea ice and deformation of a composite of ice types, has upon the geophysical-scale rheology. In addition, since the approach presented captures the anisotropy of sub-100 km dynamics, the rheology obtained would be expected to be more suitable at fine resolutions (e.g. 10 km) than the currently implemented isotropic, viscous-plastic rheology (Hibler, 1979).

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